**Blouch SBR1 Model Validation Results: 2023**

I evaluated the model’s performance on data I simulated, with scripts to reproduce simulated data available in the Simulation Code folder on github.com.

Data was simulated mirroring the causal model that generated the data. All data was simulated on a randomly sampled set of tip species from the 10K trees primate phylogeny. This data was then analyzed using the requisite blouch model, and parameter values were compared to the true parameter values. I used the set.seed() function for reproducibility and set this value to 10.

For each model I tested data simulated with the following features:

1. Short, medium, and long half-lives
2. Presence of measurement error
3. Increasing number of predictors including measurement error

For each model that accounts for measurement error, measurement error was added to the X and Y variables by simulating from a random normal distribution with mean=0 and standard deviation 0.01.

For the models that test hypotheses of regime placement, in addition to varying the parameters above these models tested:

1. Increasing number of regimes

All runs were using two chains were 2000 interation per chain.

**Direct Effect Model without Measurement Error**

Short half-life (hl=0.1)



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.16 | 0 | 0.08 | 0.06 | 0.1 | 0.14 | 0.19 | 0.37 | 964 | 1 |
| vy | 0.15 | 0 | 0.05 | 0.09 | 0.12 | 0.14 | 0.17 | 0.28 | 871 | 1 |
| alpha | 2 | 0 | 0.09 | 1.82 | 1.95 | 2.01 | 2.07 | 2.18 | 1159 | 1 |
| beta[1] | 0.27 | 0 | 0.02 | 0.22 | 0.25 | 0.27 | 0.28 | 0.31 | 1243 | 1 |
| sigma2\_y | 1.55 | 0.02 | 0.61 | 0.78 | 1.14 | 1.42 | 1.8 | 2.97 | 1533 | 1 |

Medium half-life (hl = 0.25)



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.28 | 0 | 0.13 | 0.12 | 0.19 | 0.25 | 0.33 | 0.64 | 735 | 1 |
| vy | 0.14 | 0 | 0.05 | 0.08 | 0.11 | 0.13 | 0.16 | 0.26 | 708 | 1 |
| alpha | 1.93 | 0 | 0.1 | 1.73 | 1.87 | 1.94 | 2 | 2.14 | 1352 | 1 |
| beta[1] | 0.26 | 0 | 0.02 | 0.22 | 0.25 | 0.26 | 0.28 | 0.31 | 1424 | 1 |
| sigma2\_y | 0.75 | 0.01 | 0.23 | 0.44 | 0.59 | 0.71 | 0.87 | 1.33 | 1830 | 1 |

Long half-life (hl = 0.75)



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.66 | 0.01 | 0.35 | 0.27 | 0.43 | 0.57 | 0.79 | 1.54 | 706.00 | 1.00 |
| vy | 0.12 | 0.00 | 0.05 | 0.06 | 0.08 | 0.10 | 0.14 | 0.26 | 686.00 | 1.00 |
| alpha | 2.03 | 0.00 | 0.10 | 1.83 | 1.96 | 2.03 | 2.10 | 2.23 | 1119.00 | 1.00 |
| beta[1] | 0.23 | 0.00 | 0.02 | 0.19 | 0.22 | 0.23 | 0.24 | 0.26 | 1344.00 | 1.00 |
| sigma2\_y | 0.26 | 0.00 | 0.07 | 0.16 | 0.22 | 0.25 | 0.30 | 0.43 | 1851.00 | 1.00 |

**Direct Effect Model with Measurement Error**

Short half-life (hl=0.1)



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.17 | 0 | 0.09 | 6.00% | 11% | 15% | 20% | 39.00% | 953 | 1 |
| vy | 0.16 | 0 | 0.05 | 0.09 | 0.12 | 0.15 | 0.18 | 0.29 | 1071 | 1 |
| alpha | 1.98 | 0 | 0.11 | 1.75 | 1.91 | 1.98 | 2.04 | 2.18 | 1568 | 1 |
| beta[1] | 0.27 | 0 | 0.02 | 0.22 | 0.25 | 0.27 | 0.29 | 0.32 | 2397 | 1 |
| sigma2\_y | 1.5 | 0.02 | 0.7 | 0.78 | 1.09 | 1.36 | 1.72 | 3.07 | 1400 | 1 |

Medium half-life (hl=0.25)

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.3 | 0 | 0.15 | 0.12 | 0.2 | 0.26 | 0.35 | 0.67 | 1301 | 1 |
| vy | 0.14 | 0 | 0.05 | 0.08 | 0.11 | 0.13 | 0.17 | 0.28 | 1277 | 1 |
| alpha | 1.89 | 0 | 0.13 | 1.63 | 1.81 | 1.89 | 1.97 | 2.14 | 2039 | 1 |
| beta[1] | 0.27 | 0.00 | 0.03 | 0.22 | 0.25 | 0.27 | 0.29 | 0.32 | 2596.00 | 1 |
| sigma2\_y | 0.73 | 0.01 | 0.23 | 0.40 | 0.57 | 0.69 | 0.85 | 1.29 | 1848.00 | 1 |

Long half-life (hl=0.75)



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.71 | 0.01 | 0.39 | 29.00% | 46% | 61% | 85% | 174.00% | 1171 | 1 |
| vy | 0.12 | 0 | 0.06 | 6.00% | 8% | 11% | 14% | 28.00% | 1137 | 1 |
| alpha | 2 | 0 | 0.12 | 1.76 | 1.93 | 2 | 2.08 | 2.24 | 2509 | 1 |
| beta[1] | 0.23 | 0 | 0.02 | 0.2 | 0.22 | 0.23 | 0.24 | 0.26 | 3532 | 1 |
| sigma2\_y | 0.25 | 0 | 0.07 | 0.15 | 0.2 | 0.24 | 0.29 | 0.41 | 2041 | 1 |

**Adaptive Model without Measurement Error**

Short half-life (hl=0.1)

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|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.13 | 0 | 0.06 | 0.05 | 0.09 | 0.11 | 0.15 | 0.28 | 718 | 1 |
| vy | 0.14 | 0 | 0.04 | 0.08 | 0.11 | 0.13 | 0.16 | 0.23 | 912.00 | 1 |
| alpha | 1.92 | 0.00 | 0.08 | 1.75 | 1.86 | 1.92 | 1.97 | 2.09 | 1084.00 | 1 |
| beta[1] | 0.25 | 0 | 0.04 | 0.19 | 0.22 | 0.24 | 0.26 | 0.34 | 767 | 1 |
| beta\_e[1] | 0.2 | 0 | 0.02 | 0.16 | 0.19 | 0.2 | 0.21 | 0.24 | 1497 | 1 |
| sigma2\_y | 1.72 | 0.02 | 0.68 | 0.87 | 1.26 | 1.57 | 2.01 | 3.39 | 1204 | 1 |

Medium half-life (hl=0.25)

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|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.28 | 0 | 0.12 | 0.12 | 0.2 | 0.26 | 0.34 | 0.59 | 698 | 1.00 |
| vy | 0.14 | 0 | 0.04 | 0.08 | 0.11 | 0.13 | 0.16 | 0.25 | 810 | 1.01 |
| alpha | 1.94 | 0.00 | 0.10 | 1.74 | 1.87 | 1.94 | 2.01 | 2.14 | 1039.00 | 1.00 |
| beta[1] | 0.26 | 0.00 | 0.07 | 0.16 | 0.22 | 0.25 | 0.29 | 0.42 | 732.00 | 1.00 |
| beta\_e[1] | 0.16 | 0.00 | 0.02 | 0.12 | 0.15 | 0.16 | 0.18 | 0.21 | 1639.00 | 1.00 |
| sigma2\_y | 0.76 | 0.01 | 0.24 | 0.43 | 0.59 | 0.71 | 0.87 | 1.34 | 1289.00 | 1.00 |

Long half-life (hl=0.75)

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.72 | 0.01 | 0.29 | 0.34 | 0.52 | 0.67 | 0.86 | 1.45 | 699 | 1 |
| vy | 0.13 | 0.00 | 0.05 | 0.06 | 0.09 | 0.12 | 0.15 | 0.24 | 685.00 | 1 |
| alpha | 2.06 | 0.00 | 0.11 | 1.84 | 1.99 | 2.06 | 2.13 | 2.28 | 1148.00 | 1 |
| beta[1] | 0.27 | 0.00 | 0.09 | 0.14 | 0.21 | 0.26 | 0.32 | 0.48 | 752.00 | 1 |
| beta\_e[1] | 0.10 | 0.00 | 0.02 | 0.06 | 0.09 | 0.10 | 0.11 | 0.13 | 2078.00 | 1 |
| sigma2\_y | 0.25 | 0.00 | 0.06 | 0.16 | 0.21 | 0.24 | 0.29 | 0.40 | 1805.00 | 1 |

**Adaptive model with measurement error**

Short half-life (hl=0.1)



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.13 | 0.00 | 0.06 | 0.05 | 0.09 | 0.11 | 0.15 | 0.28 | 1588.00 | 1.00 |
| vy | 0.14 | 0.00 | 0.04 | 0.08 | 0.11 | 0.13 | 0.16 | 0.23 | 1764.00 | 1.00 |
| alpha | 1.92 | 0.00 | 0.08 | 1.77 | 1.87 | 1.92 | 1.97 | 2.08 | 3026.00 | 1.00 |
| beta[1] | 0.25 | 0.00 | 0.04 | 0.19 | 0.22 | 0.24 | 0.27 | 0.35 | 1901.00 | 1.00 |
| beta\_e[1] | 0.20 | 0.00 | 0.02 | 0.16 | 0.19 | 0.20 | 0.22 | 0.24 | 2572.00 | 1.00 |
| sigma2\_y | 1.72 | 0.02 | 0.70 | 0.84 | 1.25 | 1.57 | 1.99 | 3.50 | 2183.00 | 1.00 |

Medium half-life (hl=0.25)



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.29 | 0.00 | 0.13 | 0.12 | 0.20 | 0.26 | 0.35 | 0.60 | 1379.00 | 1.00 |
| vy | 0.15 | 0.00 | 0.05 | 0.08 | 0.11 | 0.14 | 0.17 | 0.27 | 1468.00 | 1.00 |
| alpha | 1.94 | 0.00 | 0.11 | 1.74 | 1.87 | 1.94 | 2.00 | 2.17 | 3509.00 | 1.00 |
| beta[1] | 0.26 | 0.00 | 0.07 | 0.16 | 0.21 | 0.25 | 0.30 | 0.43 | 1585.00 | 1.00 |
| beta\_e[1] | 0.16 | 0.00 | 0.02 | 0.11 | 0.15 | 0.16 | 0.18 | 0.21 | 2689.00 | 1.00 |
| sigma2\_y | 0.76 | 0.00 | 0.24 | 0.43 | 0.59 | 0.71 | 0.86 | 1.33 | 2485.00 | 1.00 |

Long half-life (hl=0.75)



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.72 | 0.01 | 0.30 | 0.34 | 0.52 | 0.66 | 0.86 | 1.44 | 1200.00 | 1.00 |
| vy | 0.12 | 0.00 | 0.05 | 0.06 | 0.09 | 0.12 | 0.15 | 0.24 | 1121.00 | 1.00 |
| alpha | 2.06 | 0.00 | 0.11 | 1.83 | 1.98 | 2.06 | 2.13 | 2.27 | 3196.00 | 1.00 |
| beta[1] | 0.27 | 0.00 | 0.09 | 0.15 | 0.21 | 0.26 | 0.32 | 0.48 | 1312.00 | 1.00 |
| beta\_e[1] | 0.10 | 0.00 | 0.02 | 0.06 | 0.09 | 0.10 | 0.11 | 0.13 | 2408.00 | 1.00 |
| sigma2\_y | 0.25 | 0.00 | 0.07 | 0.15 | 0.20 | 0.24 | 0.29 | 0.41 | 1983.00 | 1.00 |

**Direct Effect Model with Measurement Error and Multiple Predictors**

Two predictors



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.18 | 0.00 | 0.10 | 0.06 | 0.11 | 0.15 | 0.22 | 0.43 | 729.00 | 1.00 |
| vy | 0.11 | 0.00 | 0.04 | 0.06 | 0.08 | 0.10 | 0.12 | 0.20 | 720.00 | 1.00 |
| alpha | 2.00 | 0.00 | 0.10 | 1.80 | 1.94 | 2.00 | 2.05 | 2.19 | 1422.00 | 1.00 |
| beta[1] | 0.42 | 0.00 | 0.09 | 0.21 | 0.36 | 0.42 | 0.48 | 0.58 | 1224.00 | 1.00 |
| beta[2] | 0.09 | 0.00 | 0.07 | 0.00 | 0.04 | 0.08 | 0.14 | 0.27 | 1100.00 | 1.00 |
| sigma2\_y | 0.96 | 0.01 | 0.42 | 0.49 | 0.70 | 0.88 | 1.12 | 1.97 | 1749.00 | 1.00 |

Three predictors



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.12 | 0 | 0.06 | 0.04 | 0.07 | 0.1 | 0.14 | 0.27 | 832 | 1 |
| vy | 0.12 | 0.00 | 0.04 | 0.07 | 0.09 | 0.11 | 0.14 | 0.22 | 681.00 | 1.00 |
| alpha | 2.08 | 0.00 | 0.08 | 1.91 | 2.04 | 2.08 | 2.13 | 2.22 | 850.00 | 1.00 |
| beta[1] | 0.48 | 0.00 | 0.10 | 0.28 | 0.41 | 0.48 | 0.55 | 0.68 | 1897.00 | 1.00 |
| beta[2] | 0.37 | 0.00 | 0.14 | 0.08 | 0.27 | 0.37 | 0.46 | 0.63 | 903.00 | 1.00 |
| beta[3] | 0.17 | 0.00 | 0.11 | 0.01 | 0.08 | 0.16 | 0.25 | 0.44 | 1354.00 | 1.00 |
| sigma2\_y | 1.66 | 0.02 | 0.71 | 0.80 | 1.18 | 1.49 | 1.93 | 3.57 | 1528.00 | 1.00 |

**Adaptive Model with Measurement Error and Multiple Traits**

**Two traits**

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|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.17 | 0.00 | 0.10 | 0.06 | 0.11 | 0.15 | 0.21 | 0.41 | 1547.00 | 1.00 |
| vy | 0.10 | 0.00 | 0.04 | 0.06 | 0.08 | 0.10 | 0.12 | 0.19 | 1582.00 | 1.00 |
| alpha | 1.99 | 0.00 | 0.10 | 1.79 | 1.93 | 1.99 | 2.05 | 2.17 | 2118.00 | 1.00 |
| beta[1] | 0.41 | 0.00 | 0.06 | 0.29 | 0.37 | 0.41 | 0.45 | 0.54 | 2381.00 | 1.00 |
| beta[2] | 0.08 | 0.00 | 0.05 | 0.00 | 0.04 | 0.07 | 0.11 | 0.20 | 1960.00 | 1.00 |
| sigma2\_y | 0.96 | 0.01 | 0.40 | 0.48 | 0.70 | 0.89 | 1.12 | 1.91 | 1974.00 | 1.00 |

Three traits



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Summary Table | mean | se\_mean | sd | 2.50% | 25% | 50% | 75% | 97.50% | n\_eff | Rhat |
| hl | 0.21 | 0.00 | 0.09 | 0.09 | 0.15 | 0.20 | 0.26 | 0.44 | 2239.00 | 1.00 |
| vy | 0.11 | 0.00 | 0.04 | 0.05 | 0.08 | 0.10 | 0.12 | 0.21 | 1857.00 | 1.00 |
| alpha | 1.99 | 0.00 | 0.10 | 1.80 | 1.92 | 1.99 | 2.05 | 2.17 | 3858.00 | 1.00 |
| beta[1] | 0.39 | 0.00 | 0.11 | 0.19 | 0.32 | 0.39 | 0.46 | 0.62 | 2862.00 | 1.00 |
| beta[2] | 0.20 | 0.00 | 0.10 | 0.02 | 0.13 | 0.19 | 0.26 | 0.42 | 1406.00 | 1.00 |
| beta[3] | 0.09 | 0.00 | 0.07 | 0.00 | 0.04 | 0.08 | 0.14 | 0.26 | 2546.00 | 1.00 |
| beta\_e[1] | 0.28 | 0.00 | 0.07 | 0.13 | 0.23 | 0.28 | 0.33 | 0.43 | 2713.00 | 1.00 |
| beta\_e[2] | 0.14 | 0.00 | 0.07 | 0.01 | 0.09 | 0.14 | 0.19 | 0.28 | 1321.00 | 1.00 |
| beta\_e[3] | 0.07 | 0.00 | 0.05 | 0.00 | 0.03 | 0.06 | 0.09 | 0.18 | 2551.00 | 1.00 |
| sigma2\_y | 0.75 | 0.01 | 0.31 | 0.35 | 0.54 | 0.69 | 0.88 | 1.54 | 1935.00 | 1.00 |